A Life in Mathematics

Generalized Functions, Microlocal Analysis,
PDEs and Dynamical Systems

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Conference in memory of Todor V. Gramchev

Book of Abstracts

Todor Vassiliev Gramchev
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Contents

THE SCHWARTZ KERNEL THEOREM AND DISTRIBUTIONS OF FINITE ORDER

NENAD ANTONIĆ, MARKO ERCEG, MARIN MIŠUR

We define distributions of anisotropic order, and establish their immediate properties. The central result is the Schwartz kernel theorem for such distributions, which represents continuous operators from $\mathrm{C}_c^l(X)$ to $\mathcal{D}'_m(Y)$ by kernels, which are distributions of order l in x, but higher, though still finite order $d(m+2)$ in y. Standard proofs of that theorem all depend on the reflexivity and Montel property of considered spaces, which is not the case here.

This result allows us to obtain more precise results on H-distributions, a recently introduced generalisation of H-measures [4], which are, therefore, distributions of order 0 (i.e. Radon measures) in $x \in \mathbf{R}^d$, and of finite order in $\xi \in \mathrm{S}^{d-1}$. Variants of H-distributions have been successfully applied to problems in velocity averaging (Lazar-Mitrović 2012) and compensated compactness with variable coefficients (Mišur-Mitrović 2015). Extension to Sobolev space setting is given in (Aleksić-Pilipović-Vojnović 2016).

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STOCHASTIC HYPERBOLIC EQUATIONS ON $\mathbb{R}^n$

ALESSIA ASCANELLI, SANDRO CORIASCO, AND ANDRÉ SÜSS

We study stochastic partial differential equations of the form

$$L(t,x,\partial_t,\partial_x)u(t,x) = \gamma(t,x,u(t,x)) + \sigma(t,x,u(t,x))\dot{\Xi}(t,x), \tag{1}$$

where $(t,x) \in [0,T] \times \mathbb{R}^n$, L is a partial differential operator of hyperbolic type, γ, σ are real valued suitable functions and Ξ is a gaussian random noise.
We provide conditions on the operator L, on the Cauchy data and on the stochastic term (namely, on the spectral measure associated to Ξ) to get the existence of a unique stochastic process u, (mild) solution of (1) in a suitable class of distributions.
More precisely, for linear equations we look for a *random-field solution*, i.e. a solution defined as a random variable for each (t,x); for semilinear equations we look for a *function-valued solution*, i.e. an Hilbert-space valued random element in the temporal argument.
These results have been recently obtained in [1] and [2].

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WEYL'S LAW
FOR TENSOR PRODUCTS OF PSEUDODIFFERENTIAL OPERATORS

UBERTINO BATTISTI, MASSIMO BORSERO, SANDRO CORIASCO

We present some results about the Weyl's law for tensor product of pseudodifferential operators. We consider operators of the form

$$A_1 \otimes A_2 \otimes \ldots \otimes A_r$$

where A_i are positive elliptic selfadjoint pseudodifferential operators acting either on closed manifolds M_i or global Shubin operators acting on $\mathbb{R}^{n_i}$. We can obtain not only the leading term but also the second term in the asymptotic expansion.

In the case of tensor product of two pseudodifferential operators we can provide explicit examples which show the sharpness of our estimates.

Our results are a consequence of a fine analysis of the poles of the spectral ζ-function.

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REGULARITY OF OPERATORS VIA WIGNER TYPE TRANSFORMS

CHIARA BOITI, DAVID JORNET, AND ALESSANDRO OLIARO

A linear operator $A: \mathcal{S}' \to \mathcal{S}'$ is said to be *regular* if

$$Au \in \mathcal{S} \;\Rightarrow\; u \in \mathcal{S}, \qquad \forall u \in \mathcal{S}'.$$

In [1] we study regularity of linear partial differential operators $A(x, D)$ with polynomial coefficients (the symbol is then a polynomial $a(x, \xi)$ of order m).

A sufficient condition for regularity is *global hypoellipticity* in the sense of Shubin: there exist $m' \leq m$, $\rho \in (0, 1]$, $c, C, B > 0$ such that

$$\begin{aligned} |a(x,\xi)| &\geq c\langle (x,\xi)\rangle^{m'} \\ |\partial_x^\alpha \partial_\xi^\beta a(x,\xi)| &\leq C|a(x,\xi)|\langle (x,\xi)\rangle^{-\rho(|\alpha|+|\beta|)}, \qquad \forall \alpha, \beta \in \mathbb{N}_0^n \end{aligned}$$

for $|(x, \xi)| \geq B$, where $\langle (x, \xi)\rangle := \sqrt{1 + |x|^2 + |\xi|^2}$.

However, this condition is far from being necessary for regularity and the question of proving regularity for non-global hypoelliptic operators is not trivial, in general.

In [1] we study regularity of linear p.d.o. with polynomial coefficients by using a Wigner type transform of the type

$$\mathrm{Wig}[w](x, y) := \int e^{-it\cdot y} w\left(x + \frac{1}{2}t, x - \frac{1}{2}t\right) dt,$$

which is an invertible operator from $\mathcal{S}$ to $\mathcal{S}$ and from $\mathcal{S}'$ to $\mathcal{S}'$, and then Cohen classes of the form

$$Q[w] := \sigma * \mathrm{Wig}[w]$$

for a kernel $\sigma \in \mathcal{S}'$.

The idea is to transform a l.p.d.o. B with polynomial coefficients into another l.p.d.o. $\tilde{B}$ with polynomial coefficients by a formula of the form

$$Q[Bw] = \tilde{B}Q[w],$$

and to prove that, under suitable assumptions on the kernel σ, the regularity is preserved by such a transformation, so that if we start from a global hypoelliptic operator B (which is therefore regular), we find in general a non-global hypoelliptic operator $\tilde{B}$ which is still regular. These results enable us to find classes of regular (but not hypoelliptic) operators, and these classes are quite large because of the freedom in the choice of the kernel σ.

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ON GENERALIZED ULTRADISTRIBUTIONS

CHIKH BOUZAR

Sobolev-Schwartz distributions [1] and [2] have natural extensions the ultradistributions in the sense of [3] and [4]. Gevrey ultradistributions are a particular, but important, case of ultradistributions.

An algebra of generalized functions containing the space of distributions have been introduced and studied in [5], this algebra gives a solution to the problem of multiplication of distributions.

In that same vein, the multiplication of ultradistributions naturally posed the problem of the existence of algebras of generalized functions containing spaces of ultradistributions.

In this work, we first give a review of works dealing with algebras of generalized ultradistibutions and then we show the role of the paper of T. Gramchev [6] in the construction of such algebras.

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COMPARISON PRINCIPLES FOR WEAKLY-COUPLED ELLIPTIC AND PARABOLIC SYSTEMS

GEORGI BOYADJIEV

The validity of comparison principle for weakly coupled quasi-linear systems of elliptic and parabolic systems is considered. In the case of cooperative systems the results concern reaction-diffusion systems, while for non-cooperative systems the coefficients are smooth.

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PERTURBATIONS OF GLOBALLY HYPOELLIPTIC INVARIANT OPERATORS ON SMOOTH MANIFOLDS

ALEXANDRE KIRILOV AND FERNANDO DE ÁVILA SILVA

We present recent results on the investigation of globally hypoelliptic perturbations of the operators in the class

$$L \doteq D_t + C(x, D_x), (t, x) \in \mathbb{T} \times M, \tag{1}$$

where $\mathbb{T} = \mathbb{R}/2\pi\mathbb{Z}$ stands for the flat torus, M is a closed smooth manifold and $C(x, D_x)$ is a strongly invariant operator with respect to some elliptic operator $E = E(x, D_x)$ defined on the manifold M.

More specifically, we ask the following: if L is globally hypoelliptic and $R(x, D_x)$ is another strongly invariant operator, with respect to E, then is the operator

$$P \doteq L + R(x, D_x), (t, x) \in \mathbb{T} \times M \tag{2}$$

globally hypoelliptic?

Under the assumption that $C(x, D_x)$ and $R(x, D_x)$ are invariant with respect to an elliptic operator $E(x, D_x)$ defined on M, we show that the global hypoellipticity properties of P and L can be studied in view of the behavior at infinity of the eigenvalues of the matrix representations of the restrictions

$$C(x, D_x) : E_j \to E_j \quad \text{and} \quad R(x, D_x) : E_j \to E_j,$$

on the eigenspaces E_j of $E(x, D_x)$.

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THE BARGMANN TRANSFORM AND POWERS OF HARMONIC OSCILLATOR ON GELFAND-SHILOV SUBSPACES

CARMEN FERNÁNDEZ

The counter image of entire functions of exponential type under the Bargmann transform consists of those consists of all $f \in \mathscr{S}(\mathbf{R}^d)$ such that their Hermite series expansions are given by

$$f = \sum_{\alpha \in \mathbf{N}^d} c_\alpha(f) h_\alpha,$$

where

$$|c_\alpha(f)| \leq \frac{C r^{|\alpha|}}{\sqrt{\alpha!}},$$

for some constants $r > 0$ and $C > 0$. We present a characterization of these functions in terms of estimates of powers of the harmonic oscillator $H = |x|^2 - \Delta$. We also consider the Pilipović spaces $\boldsymbol{\mathcal{S}}_s(\mathbf{R}^d)$ and $\boldsymbol{\Sigma}_s(\mathbf{R}^d)$ when $0 < s < 1/2$ and deduce their images under the Bargmann transform.

Joint work with A. Galbis (València) and J. Toft (Växjö).

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COMPACT SOLITARY WAVES

GIUSEPPE GAETA

Back in 2006, Todor – together with Sebastian Walcher and myself – wrote a paper about *compactons*, i.e. solitary waves with strictly compact support, arising in nonlinear chains with non-smooth potential [1, 2]. I will recall this work and review some subsequent developments and applications, in particular in continuum mechanics [3, 4] and in field theory [5].

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FRACTIONAL H-MEASURES AND TRANSPORT PROPERTY

MARKO ERCEG AND IVAN IVEC

Microlocal defect functionals (H-measures, H-distributions, semiclassical measures etc.) are objects which determine, in some sense, the lack of strong compactness for weakly convergent L^p sequences. H-measures are suitable to treat problems where all partial derivatives are of the same order [4]. Recently, parabolic H-measures are introduced in order to treat 1:2 ratio between orders of partial derivatives [1], and also fractional H-measures which treat arbitrary ratios [2, 3].

We generalise Second commutation lemmas introduced in [1] and [4] to fractional H-measures, from which we are able to derive the propagation principle for the following fourth order partial differential equation:

$$iu_t + (a(x)u_{xx})_{xx} = f.$$

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SPACES OF ULTRADISTRIBUTIONS ON $\mathbb{R}^d_+$ WITH APPLICATIONS TO PSEUDO-DIFFERENTIAL OPERATORS WITH RADIAL SYMBOLS

SMILJANA JAKŠIĆ

The first part of the talk is devoted to the spaces $G^\alpha_\alpha(\mathbb{R}^d_+)$, $\alpha \geq 1$ and their dual spaces which can be described as analogous to the Gelfand-Shilov spaces and their dual spaces. The elements in $G^\alpha_\alpha(\mathbb{R}^d_+)$, $\alpha \geq 1$ and their dual spaces are characterized through the Laguerre expansions. The second part is devoted to the class of the Weyl pseudo-differential operators with radial symbols from $G^\alpha_\alpha(\mathbb{R}^d_+)$, $\alpha \geq 1$ and their dual spaces. The continuity properties of these classes of pseudo-differential operators over the Gelfand-Shilov spaces and their dual spaces are proved. In this way the classes of the Weyl pseudo-differential operators are extended to those with the radial symbols with the exponential and sub-exponential growth rate.

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GLOBAL HYPOELLIPTICITY FOR PSEUDO-DIFFERENTIAL OPERATORS ON THE TORUS

ALEXANDRE KIRILOV

We investigate the global hypoellipticity of the operator

$$L = D_t + c(t)P(D_x),\ (t,x) \in \mathbb{T}^1 \times \mathbb{T}^n, \tag{1}$$

where $c(t)$ is a complex smooth function on $\mathbb{T}^1$, and $P(D_x)$ is a pseudo-differential operator of order $m \in \mathbb{R}$ defined on $\mathbb{T}^n$, with toroidal symbol $p = p(\xi) \in S^m(\mathbb{Z}^n)$.

We say that L is globally hypoelliptic on $\mathbb{T}^1 \times \mathbb{T}^n$ (briefly GH) if the conditions $u \in \mathcal{D}'(\mathbb{T}^1 \times \mathbb{T}^n)$ and $Lu \in C^\infty(\mathbb{T}^1 \times \mathbb{T}^n)$ imply that $u \in C^\infty(\mathbb{T}^1 \times \mathbb{T}^n)$.

When $P(D_x) = D_x$ and $n = 1$, J. Hounie proved in [4] that $D_t + c(t)D_x$ is GH if and only if $\Im c(t)$ does not change sign and either $\Im c_0 \neq 0$ or $\Re c_0$ is an irrational non-Liouville number, where

$$c_0 \doteq \frac{1}{2\pi}\int_0^{2\pi} c(t)dt.$$

We recall that S. Greenfield and N. Wallach have proved in [3] that the above conditions on c_0 means that the constant coefficient operator $D_t + c_0 D_x$ is GH. Therefore, the global hypoellipticity of $D_t + c_0 D_x$ is a necessary condition for the global hypoellipticity of the operator with variable coefficients $D_t + c(t)D_x$.

We prove that this necessity remains valid for any pseudo-differential operator $P(D_x)$ defined on the n-dimensional torus, that is, if the operator L defined in (1) is GH then the constant coefficient operator

$$L_0 = D_t + c_0 P(D_x),$$

is also GH.

We also show that the global hypoellipticity of L_0 and the control of the sign of the imaginary part of the functions

$$t \in \mathbb{T}^1 \mapsto c(t)p(\xi),\ \xi \in \mathbb{Z}^n,$$

for sufficiently large $|\xi|$, are sufficient conditions to the global hypoellipticity of L.

This is a joint work with Fernando de Ávila Silva, Rafael Borro Gonzalez and Cleber de Medeira.

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SHAPE DERIVATIVE METHOD FOR OPTIMAL DESIGN IN CONDUCTIVITY PROBLEM

PETAR KUNŠTEK AND MARKO VRDOLJAK

We consider the conductivity problem in an annulus $\Omega \subseteq \mathbf{R}^d$:

$$\begin{aligned} -\mathrm{div}(\mathbf{A}\nabla u) &= f \\ u &\in \mathrm{H}_0^1(\Omega)\,, \end{aligned}$$

where the conductivity matrix $\mathbf{A}$ is of the form $\mathbf{A} = \chi\alpha\mathbf{I} + (1-\chi)\beta\mathbf{I}$, with a characteristic function χ representing the region occupied by the first phase. The optimal design problem deals with maximization of the energy functional $I(\chi) = \int_\Omega f u \, d\mathbf{x}$, over the set of all measurable characteristic functions χ satisfying the condition $\int_\Omega \chi \, d\mathbf{x} = q_\alpha$, which prescribes the amounts of given phases.

The homogenisation method proved to be well suited for treatment of optimal design for elliptic problems (in modelling both conductivity and elasticity), first for the theoretical questions on proper relaxation, but also as a starting point for application of classical methods of calculus of variations leading to necessary conditions of optimality.

We shall consider the problem with a constant right-hand side f. The interesting result is that on a simply connected open set Ω, with smooth connected boundary, the classical solution appears only if Ω is a ball.

If Ω is a ball, in order to maximize the energy the better conductor should be placed inside a smaller (concentric) ball, whose radius can easily be calculated from the constraint on given amounts of materials. By analysing the optimality conditions, we are able to show that in the case of annulus, the solution is also unique, classical and radial. Depending on the amounts of given materials, we find two possible optimal configurations. If the amount of the first phase is less than some critical value, then the better conductor should be placed in an outer annulus. Otherwise, the optimal configuration consists of an annulus with the better conductor, surrounded by two annuli of the worse conductor. The same holds true in two and three dimensions.

We present the implementation of shape derivative method for numerical solution on this example. The method shows good convergence properties toward the solution which was theoretically obtained by analysing optimality conditions.

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ASYMPTOTICS FOR THE HYPERBOLIC UMBILIC CAUSTIC IN GEVREY SPACES

ALBERTO LOVISON AND FRANCO CARDIN

In this talk we will present a rigorous study of the oscillatory integral related to the celebrated hyperbolic umbilic caustic. We propose seemingly novel precise asymptotic expansions and precise quantitative estimates for the error terms. We focus in particular in the symmetric case, where we obtain an explicit description of the Morse integration domains.

These estimates are obtained by means of a systematic strategy devised by Todor Gramchev using Fubini decomposition and distributional derivatives in Gevrey spaces [3].

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CONVOLUTION AND PRODUCT OF ULTRADISTRIBUTIONS IN SEQUENTIAL APPROACH

SNJEZANA MAKCIMOVIĆ

We introduce and analyze the existence of product and convolutions of ultradistributions using a sequential approach to ultradistribution spaces.

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ESTIMATES FOR THE BLOW-UP TIME TO SOLUTIONS IN CHEMOTAXIS SYSTEMS WITH A SOURCE TERM

MONICA MARRAS

We investigate a class of parabolic-parabolic Keller-Segel type system in a bounded domain in $\mathbb{R}^N$, with $N = 2, 3$, under different boundary conditions, with time dependent coefficients, a nonlinear cross diffusion and a positive source term. The solutions may blow up in finite time T: we discuss different methods to derive explicit estimates for the blow-up time.

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GENERALIZED FUNCTIONS ON THE CLOSURE OF AN OPEN SET. APPLICATION TO SOME PROBLEMS OF UNIQUENESS

VICTOR DÉVOUÉ, JEAN-ANDRÉ MARTI, HANS VERNAEVE AND JASSON VINDAS

The space $\mathcal{O}_M(\mathbb{R}^n)$ of slowly increasing functions, endowed by the family of semi-norms $(p_{\varphi,\alpha})_{(\varphi,\alpha)\in\mathcal{S}(\mathbb{R}^n)\times\mathbb{N}^n}$, becomes a topological algebra used in [2] to define the generalized algebra $\mathcal{G}_{\mathcal{O}_M}(\mathbb{R}^n)$ (which differs from $\mathcal{G}_\tau(\mathbb{R}^n)$). It is very useful to prove the uniqueness of some linear characteristic Cauchy problem studied in [1].

But in nonlinear cases, we cannot obtain the result without replacing $\mathbb{R}^n$ by a smaller closed set. When Ω is a convex open set in $\mathbb{R}^n$, we prove that $\mathcal{O}_M\left(\overline{\Omega}\right)$, with the topology deducted from that of $\mathcal{O}_M(\mathbb{R}^n)$ by replacing $\mathcal{S}(\mathbb{R}^n)$ by $\mathcal{S}\left(\overline{\Omega}\right)$, becomes also a locally convex algebra. Now, we define the generalized algebra $\mathcal{G}_{\mathcal{O}_M}\left(\overline{\Omega}\right)$ as the quotient algebra $\mathcal{M}_{\mathcal{O}_M}\left(\overline{\Omega}\right)/\mathcal{N}_{\mathcal{O}_M}\left(\overline{\Omega}\right)$. When Ω is unbounded, it is given an alternative representation of $\mathcal{N}_{\mathcal{O}_M}\left(\overline{\Omega}\right)$ leading to a point-value characterization ([4], [3]) of elements in $\mathcal{G}_{\mathcal{O}_M}\left(\overline{\Omega}\right)$. There is the toolbox to obtain the uniqueness for nonlinear differential problems as some paradigmatic characteristic Cauchy ones [5]

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EXTENSION OF CORDES' AND TARTAR'S RESULTS ON COMPACTNESS OF COMMUTATORS

NENAD ANTONIĆ, MARIN MIŠUR, AND DARKO MITROVIĆ

We generalise results on compactness of commutators of multiplication and Fourier multiplier operators by H. O. Cordes (1975) and L. Tartar (1990) in several directions with respect to the smoothness of multiplication function and by replacing the Fourier multiplier operator by a more general pseudodifferential operator. Our prime motivation has been a particular case known as the First commutation lemma – the basic tool for defining H-measures and H-distributions. We review and improve the known results both in the standard L^2 setting, as well as for general L^p, with $1 < p < \infty$. Furthermore, we extend these results to less regular simbols.

This is joint work with Nenad Antonić and Darko Mitrović.

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INHOMOGENEOUS MICROLOCAL PROPAGATION OF SINGULARITIES IN FOURIER LEBESGUE SPACES

GIANLUCA GARELLO AND ALESSANDRO MORANDO

We present some results of continuity in weighted Fourier Lebesgue spaces for pseudodifferential operators whose symbols $a(x,\xi)$ have limited Fourier Lebesgue smoothness with respect to x and grow inhomogeneously in ξ. Local and microlocal propagation of singularities are studied, with applications to some classes of semilinear pdes. This is a joint work with G. Garello.

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COLOMBEAU ALGEBRAS WITHOUT ASYMPTOTICS

EDUARD A. NIGSCH

All constructions of Colombeau algebras so far incorporate certain asymptotic estimates for the definition of the spaces of moderate and negligible functions, the quotient of which forms the algebra. There is a certain degree of freedom in the asymptotic scale employed for these estimates; most commonly a polynomial scale is used, but there exist generalizations in several directions.

I will present a (diffeomorphism invariant, full) algebra of generalized functions which, instead of asymptotic estimates obtained by inserting appropriate test objects, employs only topological estimates on certain spaces of kernels for its definition. This is a direct generalization of the usual seminorm estimates valid for distributions and appears to be a promising concept for regularity theory of nonlinear generalized functions.

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ON A CLASS OF THIRD ORDER EQUATIONS

NICOLA ORRÙ

We consider hyperbolic equations of third order, in the variables $t, x_1, \ldots, x_d$. We suppose that the coefficients depend only on t and are analytic. We give sufficient conditions for the well-posedness of Cauchy problem. Last year Prof. Wakabayashi has obtained some results similar to ours, but the conditions on lower order terms are different from ours.

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INFINITE ORDER SOBOLEV TYPE SPACES $H^*_{A_p,\rho}(f)$

STEVAN PILIPOVIĆ, BOJAN PRANGOSKI AND JASSON VINDAS

Infinite order Sobolev type spaces $H^*_{A_p,\rho}(f)$, where the order is given by a functions f belonging to a certain class of "admissible" functions of sub-exponential (i.e. ultrapolynomial) growth will be presented. $*$ and A_p stend for Gevrey type sequences, while $\rho > 0$. When $f(x,\xi) = \langle (x,\xi)\rangle^s$, $s \in \mathbb{R}$, they reduce to the classical Sobolev spaces $H^s_\Gamma(\mathbb{R}^d)$ of order s, where Γ stends for a class of global Shubin ΨDO . $H^*_{A_p,\rho}(f)$ satisfies most of the familiar results for the classical, finite order, Sobolev spaces: independence on the choice of the generating operator, a priori estimates, duality etc.; additionally, we investigate their connection to the spaces of Gelfand-Shilov type. Furthermore, we investigate Fredholm properties of infinite order ΨDOs having hypoelliptic symbols satisfying elliptic bounds with respect to an admissible function f.

Key words and phrases: Ultradistributions, infinite order pseudo-differential operators, infinite order Sobolev spaces

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BLOW-UP SOLUTIONS FOR A CLASS OF FOURTH ORDER WAVE EQUATIONS

STELLA PIRO VERNIER

Hyperbolic problems of fourth order are models in various areas of mathematical physics, as, for instance, in the theory of vibrating plates.
Our interest is to study the blow-up phenomena and to derive a lower bound for the lifespan of the solutions to such problems.

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IN MEMORY OF TODOR V. GRAMCHEV

PETAR POPIVANOV

This talk deals with some of the first steps of Todor Gramchev in the domain of Gevrey microlocal analysis.

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SOLVABILITY AND HYPO-ELLIPTICITY FOR OPERATORS WITH INVOLUTIVE CHARACTERISTICS AND THEIR PARAMETRICES

PETAR POPIVANOV

This talk deals with microlocal properties of pseudo-differential operators with double-involutive characteristics and partially vanishing sub-principal symbols. In some cases microlocal parametrix is constructed and hypo-ellipticity properties are studied.

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EXTENDED GEVREY REGULARITY AND WAVE FRONT SETS

NENAD TEOFANOV

We give a brief historical overview of Carleman and Gevrey classes, and proceed with a review of other spaces of ultradifferentiable funcitons. Then, we introduce and study spaces of ultradifferentiable functions related to the sequences of the form $\{p^{\tau p^\sigma}\}_{p\in\mathbf{N}}$, $\tau > 0$, $\sigma \geq 1$. This includes the Gevrey type regularity when $\sigma = 1$ and $\tau > 1$, and the analytic regularity when $\sigma = \tau = 1$.

As oppose to such regularity, we consider singular directions in phase space by introducing appropriate wave-front sets. We identify the corresponding singular supports as projections of intersections/unions of wave-front sets.

By using the procedure which we call "enumeration" we compare different types of wave front sets and discuss the corresponding regularity.

Furthermore, we use the powerful approximate solution technique to prove the microlocal embedding

$$\mathrm{WF}_{0,\infty}(P(D)u) \subseteq \mathrm{WF}_{0,\infty}(u) \subseteq \mathrm{WF}_{0,\infty}(P(D)u) \cup \mathrm{Char}(P), \quad u \in \mathcal{D}'(\mathbb{R}^d),$$

where $P(D)$ is a partial differential operator with the characteristic set $\mathrm{Char}(P)$, and $\mathrm{WF}_{0,\infty}$ is the wave front set described in terms of new regularity conditions. The proof is particularly demanding when the coefficients in $P(D)$ are non-constant.

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SCHATTEN PROPERTIES, NUCLEARITY AND MINIMALITY OF PHASE SHIFT INVARIANT SPACES

JOACHIM TOFT

We extend Feichtinger's minimality property on smallest non-trivial time-frequency shift invariant Banach spaces, to the quasi-Banach case. Analogous properties are deduced for certain matrix classes.

We use these results to prove that the pseudo-differential operator $\operatorname{Op}(a)$ is a Schatten-q operator from M^∞ to M^p and r-nuclear operator from M^∞ to M^r when $a \in M^r$ when $p, q \in (0, \infty]$ and $r \in (0, 1]$ satisfy

$$\frac{1}{r} - 1 \geq \max\left(\frac{1}{p} - 1, 0\right) + \max\left(\frac{1}{q} - 1, 0\right) + \frac{1}{q}.$$

We also present extensions of these results involving weighted modulation spaces.

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CONDITIONS OF HYPERBOLICITY OF SYSTEMS WITH CONSTANT MULTIPLICITIES

JEAN VAILLANT

We state necessary and sufficient conditions of hyperbolicity C^∞ or Gevrey for systems of linear PDE with constant multiplicity; these conditions are invariant.

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EXACT SOLUTIONS OF INTEGRABLE NONLINEAR EVOLUTION EQUATIONS

CORNELIS VAN DER MEE

Many nonlinear differential equations can be solved via the Inverse Scattering Transform (IST). In this talk, after a brief introduction of the IST, we derive an explicit solution formula for many interesting evolution equations. This talk is based on joint work with F. Demontis (University of Cagliari) and various other authors.

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REGULARITY, SMALL-DIVISORS AND DIOPHANTINE APPROXIMATION

JAMES VICKERS

It is well-known that the presence of resonances can lead to a lack of stability in Hamiltonian systems. Indeed, due to the presence of "small divisors", one needs to ensure that the frequencies are not even close to resonance in order to establish the convergence of solutions. For example in the classical KAM theorem, which demonstrates the stability of integrable Hamiltonian systems, the frequencies are required to satisfy a diophantine inequality which ensures integer linear combinations of the frequencies are bounded away from zero. What is less well-known is that similar diophantine conditions arise when considering global solvability and regularity of PDEs. Gramchev looked at this issue in a number of papers and in [1] he looked at global hypoellipticity and Gevrey regularity when vector fields on the torus are perturbed by PDOs.
In both of the examples above the method of proof involves making a change of variable to bring the differential operator into a suitable normal form. In this talk I will look at how the diffeomorphism group (or a suitable subgroup of it) acts on the differential operator and apply an infinite-dimensional inverse function theorem to give conditions under which one can bring the operator into a suitable normal form. In particular I will show how the diophantine inequality arises from the conditions required to apply the inverse function theorem. I will then show how the work of Gramchev on normal forms and regularity builds on some previous classical results and how it relates to some more recent work in the area.

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NEW DEVELOPMENTS IN THE NON-LINEAR THEORY OF GENERALIZED FUNCTIONS: OPTIMAL EMBEDDINGS OF ULTRADISTRIBUTIONS AND HYPERFUNCTIONS

JASSON VINDAS

In this talk we give an overview of various recent developments concerning the possibility to construct optimal embeddings of ultradistributions and hyperfunctions into algebras of generalized functions. We mention that T. Gramchev was the first to point out the importance of a non-linear theory for ultradistributions in his pioneer work [2], while the corresponding question for hyperfunctions was posed by M. Oberguggenberger [3, p. 286, Prob. 27.2].

Optimality of the embedding here refers to the preservation of the multiplication of ultradifferentiable functions avoiding any "loss of regularity"; in the hyperfunction case this means preserving the multiplication of real analytic functions. The construction of such optimal embeddings was up to now an important question. Our main goal is then to present a solution to the latter question. The hyperfunction and quasianalytic cases are much more difficult to deal with; in particular, their analysis requires to investigate the solvability of the Cousin problem for vector-valued quasianalytic functions [1] and more elaborate use of sheaf-theoretical arguments.

The talk is based on collaborative works with A. Debrouwere and H. Vernaeve.

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TOROIDAL PSEUDODIFFERENTIAL OPERATORS AND SPACES OF ULTRADISTRIBUTIONS ON $\mathbb{T}^n$

ĐORĐE VUČKOVIĆ

In this talk we will study a class of symbols and corresponding pseudodifferential operators of finite order on torus $\mathbb{T}^n$ that act continuously on a space of ultradistributions on $\mathbb{T}^n$, of Beurling and Roumieu type, and develop symbolic calculus for these classes.

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FOURIER INTEGRAL OPERATORS WITH QUADRATIC PHASE FUNCTIONS AND SHUBIN AMPLITUDES

MARCO CAPPIELLO, RENÉ SCHULZ, AND PATRIK WAHLBERG

Hörmander's metaplectic semigroup is defined by Schwartz kernels defined by oscillatory integrals with respect to certain quadratic phase functions. We generalize a particular case of this construction by admitting amplitudes of Shubin type. We prove that these operators can be factorized, modulo operators that are smoothing in the Schwartz sense, as a Shubin pseudodifferential operator composed with a metaplectic operator, or the other way around.

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BLOWUP OF WAVE EQUATION AND BIRKHOFF THEORY OF SOME HAMILTONIAN SYSTEM

MASAFUMI YOSHINO

In studying the blow-up of a nonlinear wave equation, one often uses the so-called self-similar solution in an asymptotic expansion. If the solution is radially symmetric, then it satisfies an ordinary differential equation called a profile equation. In my talk, we shall study the case where the profile equations are given by either a generalized Emden-Fowler equation or a nonlinear Heun equation. The linear part of the Heun equation has four regular singular points on the Riemann sphere. After elementary transformations the profile equation is written in a non autonomous nonlinear Hamiltonian system with two degrees of freedom. Our motivation of the study is to construct blowup solutions which are in the Sobolev space with negative index by virtue of Birkhoff theory. Our method of the construction of a singular solution consists of two steps. First, by making use of the symplectic transformation similar to Birkoff transformation we reduce the Hamiltonian system to a simpler form. In doing this we encounter the divergence of the symplectic transformation, which we deal with the Borel resummation method. Next, we apply the method similar to Painlevé test to the reduced Hamiltonian system in constructing a singular solution. This talk is accompanied by the brief review on the joint works with my friend professor Todor V. Gramchev.

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